DISCRETE DIFFERENTIAL GEOMETRY VIA CHECKERBOARD PATTERNS

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We develop a discrete differential geometric theory based on a recent approach using checkerboard patterns. The aim is to obtain a theory similar to classical differential geometry but applicable to discrete objects such as polygonal meshes and polyhedral surfaces. After depicting the checkerboard pattern approach we define discrete versions of derivatives, a Gauß map and a corresponding shape operator. Based on this we introduce discrete versions of conjugate and principal curvature nets. We find that discrete principal curvature nets follow the eigenvectors of the discrete shape operator. Further, the class of discrete principal nets is invariant under the Möbius transformation adapted to our setting. We then study the behaviour of discrete isothermic surfaces under Christoffel transformations. It turns out that only those isothermic meshes allow a Christoffel transformation, that are also discrete Koenigs nets introduced by Doliwa (2003).
Apart from providing a nice and consistent discrete theory, quantities defined via checkerboard patterns also show good convergence towards their smooth counterparts. This makes the checkerboard pattern approach especially interesting and suitable for applications.

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